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Fractal spacetime structure in asymptotically safe gravity

Oliver Lauscher
Institute of Theoretical Physics, University of Leipzig
Augustusplatz 10-11, D-04109 Leipzig, Germany
E-mail: lauscher@itp.uni-leipzig.de

Martin Reuter
Institute of Physics, University of Mainz
Staudingerweg 7, D-55099 Mainz, Germany
E-mail: reuter@thep.physik.uni-mainz.de

Abstract: Four-dimensional Quantum Einstein Gravity (QEG) is likely to be an asymptotically safe theory which is applicable at arbitrarily small distance scales. On sub-planckian distances it predicts that spacetime is a fractal with an effective dimensionality of 2. The original argument leading to this result was based upon the anomalous dimension of Newton's constant. In the present paper we demonstrate that also the spectral dimension equals 2 microscopically, while it is equal to 4 on macroscopic scales. This result is an exact consequence of asymptotic safety and does not rely on any truncation. Contact is made with recent Monte Carlo simulations.

Keywords: Nonperturbative Effects, Renormalization Group, Models of Quantum Gravity.

Contents

1. Introduction

Soon after the detailed investigation of Quantum Einstein Gravity [1]–[17] had begun and it had become clear that the theory is likely to be nonperturbatively renormalizable or "asymptotically safe" [8, 3, 5, 6] it was observed [3, 5] that it predicts a fractal spacetime structure at sub-planckian distances whose effective dimensionality equals 2. On the technical side, a key tool in the nonperturbative investigation of Quantum Einstein Gravity (QEG) was the effective average action and its associated exact renormalization group (RG) equation which had been developed in [18, 19] and was first applied to gravity in [1]. (For general reviews see [20, 21].)

In QEG, the effective average action $\Gamma_k[g_{\mu\nu}]$ defines an infinite set of effective field theories, valid near a variable mass scale k which is introduced as an infrared (IR) cutoff and varies between $k = 0$ and $k = \infty$ [1]. Intuitively speaking, the solution $\langle g_{\mu\nu}\rangle_k$ of the scale dependent field equation

$$\frac{\delta\Gamma_k}{\delta g_{\mu\nu}(x)}[\langle g\rangle_k] = 0 \tag{1.1}$$

can be interpreted as the metric averaged over (Euclidean) spacetime volumes of a linear extension ℓ which typically is of the order of $1/k$. Knowing the scale dependence of Γ_k, i.e. the renormalization group trajectory $k \mapsto \Gamma_k$, we can in principle follow the solution $\langle g_{\mu\nu}\rangle_k$ from the ultraviolet ($k \to \infty$) to the infrared ($k \to 0$).

It is an important feature of this approach that the infinitely many equations of (1.1), one for each scale k, are valid *simultaneously*. They all refer *to the same* physical system, the "quantum spacetime", but describe its effective metric structure on different scales. An observer using a "microscope" with a resolution $\approx k^{-1}$ will perceive the universe to be a riemannian manifold with metric $\langle g_{\mu\nu}\rangle_k$. At every fixed k, $\langle g_{\mu\nu}\rangle_k$ is a smooth classical metric. But since the quantum spacetime is characterized by the infinity of equations (1.1) with $k = 0, \dots, \infty$ it can acquire very nonclassical and in particular fractal features.

Let us describe more precisely what it means to "average" over euclidean spacetime volumes. The quantity we can freely tune is the IR cutoff scale k, and the "resolving power" of the microscope, henceforth denoted ℓ, is in general a complicated function of k. (In flat space, $\ell \approx 1/k$.) In order to understand the relationship between ℓ and k we must recall some steps from the construction of $\Gamma_k[g_{\mu\nu}]$ in ref. [1].

The effective average action is obtained by introducing an IR cutoff into the path-integral over all metrics, gauge fixed by means of a background gauge fixing condition. Even without a cutoff the resulting effective action $\Gamma[g_{\mu\nu}; \bar{g}_{\mu\nu}]$ depends on two metrics, the expectation value of the quantum field, $g_{\mu\nu}$, and the background field $\bar{g}_{\mu\nu}$. This is a standard technique, and it is well known [23] that the functional $\Gamma[g_{\mu\nu}] \equiv \Gamma[g_{\mu\nu}; \bar{g}_{\mu\nu} = g_{\mu\nu}]$ obtained by equating the two metrics can be used to generate the 1PI Green's functions of the theory.

The IR cutoff of the average action is implemented by first expressing the functional integral over all metrics in terms of eigenmodes of $\bar{D}^2$, the covariant laplacian formed with the aid of the background metric $\bar{g}_{\mu\nu}$. Then a suppression term is introduced which damps the contribution of all $-\bar{D}^2$-modes with eigenvalues smaller than k^2. Following the usual steps [20] this leads to the scale dependent functional $\Gamma_k[g_{\mu\nu}; \bar{g}_{\mu\nu}]$, and the action with one argument again obtains by equating the two metrics: $\Gamma_k[g_{\mu\nu}] \equiv \Gamma_k[g_{\mu\nu}; \bar{g}_{\mu\nu} = g_{\mu\nu}]$. It is this action which appears in (1.1). Because of the identification of the two metrics we see that it is basically the eigenmodes of $\bar{D}^2 = D^2$, constructed from the argument of $\Gamma_k[g]$, which are cut off at k^2. Since $\langle g_{\mu\nu}\rangle_k$ is the corresponding stationary point, we can say that the metric $\langle g_{\mu\nu}\rangle_k$ applies to the situation where only the quantum fluctuations of $-D^2(\langle g_{\mu\nu}\rangle_k)$ with eigenvalues larger than k^2 are integrated out. Therefore there is a complicated interrelation between the metric and the scale at which it provides an effective description: The covariant laplacian which ultimately decides about which modes are integrated out is constructed from the "on shell" configuration $\langle g_{\mu\nu}\rangle_k$, so it is k-dependent by itself already.

From these remarks it is clear now how to obtain the "resolving power" ℓ for a given k, at least in principle. We take the laplacian $-D^2(\langle g_{\mu\nu}\rangle_k)$, solve its eigenvalue problem, and then analyze in particular the properties of the eigenfunction(s) with eigenvalue k^2 (or near k^2 in the case of a discrete spectrum). Loosely speaking, this eigenfunction is the last one integrated out. As a consequence, the "averaging" scale is crucially determined by "how fast" this eigenfunction varies over spacetime. Let us assume, for instance, this eigenfunction is oscillatory with a coordinate period Δx. Then, again by using $\langle g_{\mu\nu}\rangle_k$, we compute the physical proper length this period corresponds to, and this is then what determines the resolution ℓ.

In general the eigenfunction at k^2 will have a complicated x-dependence, and therefore also the typical scales on which it varies are position dependent. Moreover, at a given point, the scale of significant variation will be direction dependent (anisotropic). Therefore the resolving power $\ell = \ell(k; x, n)$ is a complicated function in general, depending parametrically on points (x^μ) and directions (n^μ) on spacetime. It is clear that these notions can be made precise only in concrete examples and must be defined on a case by case basis.

We emphasize, however, that using the averaged metric itself to define the scale it is averaged over is not a vicious circle, but rather is exactly as it must be in a background independent approach to the quantization of gravity.

In a somewhat simplified form, the construction of a quantum spacetime within QEG can be summarized as follows. We start from a fixed RG trajectory $k \mapsto \Gamma_k$, derive its effective field equations at each k, and solve them. The resulting quantum mechanical counterpart of a classical spacetime is specified by the infinity of riemannian metrics $\{\langle g_{\mu\nu}\rangle_k \,|k = 0, \ldots, \infty\}$. While the totality of these metrics contains all physical information, the parameter k is only a book keeping device a priori. In a second step, it can be given a physical interpretation by relating it to the (proper) length scale of the averaging procedure: One constructs the laplacian $-D^2(\langle g_{\mu\nu}\rangle_k)$, diagonalizes it, looks how rapidly its k^2-eigenfunction varies, and "measures" the length of typical variations with the metric $\langle g_{\mu\nu}\rangle_k$ itself. By solving the resulting $\ell = \ell(k)$ for $k = k(\ell)$ we can in principle reinterprete the metric $\langle g_{\mu\nu}\rangle_k$ as referring to a microscope with a known position and direction dependent resolving power. The price we have to pay for the background independence is that we cannot freely choose ℓ directly but rather k only.

The first difficult step in this construction program consists in finding the RG trajectories. The running action $\Gamma_k[g_{\mu\nu}]$ is given by an exact functional RG equation [1]. In practice it is usually solved on a truncated theory space. In the Einstein-Hilbert truncation, for instance, Γ_k is approximated by a functional of the form

$$\Gamma_k[g] = (16\pi G_k)^{-1} \int d^4x \,\sqrt{g}\, \{-R(g) + 2\bar{\lambda}_k\} \tag{1.2}$$

involving a running Newton constant G_k and cosmological constant $\bar{\lambda}_k$. Their k-dependence can be obtained from the RG equation projected onto the truncation subspace. The β-functions for the dimensionless couplings $g_k \equiv k^2\, G_k$ and $\lambda_k \equiv \bar{\lambda}_k/k^2$ were first obtained in [1]. Remarkably, they turned out to possess a simultaneous zero at a non-gaussian fixed point (NGFP) (g_*, λ_*) which has just the right properties needed for the nonperturbative renormalizability of QEG along the lines of Weinberg's [22] ideas on "asymptotic safety" [8]. It was argued [3, 5, 6] that the NGFP is likely to exist also in the un-truncated, full theory and allows for the construction of a consistent and predictive microscopic theory of quantum gravity valid at arbitrarily small distances even.

One of the highly intriguing conclusions we reached in refs. [3, 5] was that the effective dimensionality of spacetime is scale dependent. It equals 4 at macroscopic distances ($\ell \gg \ell_{\rm Pl}$) but, near $\ell \approx \ell_{\rm Pl}$, it gets dynamically reduced to the value 2. For $\ell \ll \ell_{\rm Pl}$ spacetime is, in a precise sense [3], a 2-dimensional fractal.

In ref. [24] the specific form of the graviton propagator on this fractal was applied in a cosmological context. It was argued that it gives rise to a Harrison-Zeldovich spectrum of primordial geometry fluctuations, perhaps responsible for the CMBR spectrum observed today.

In refs. [24]–[32] various types of "RG improvements" were used to explore possible manifestations of the scale dependence of the gravitational parameters.

Along a quite different line of investigation, considerable progress has been made recently towards defining a quantum theory of gravity as the continuum limit of a discrete model of statistical mechanics. Performing comprehensive Monte Carlo simulations within the framework of causal (lorentzian) triangulated geometries [33], Ambjørn, Jurkiewicz and Loll [34]–[36] collected strong evidence indicating that these models can describe universes which are extended both in space and time and are 4-dimensional on large scales. In particular the above authors "measured" numerically the spectral and Hausdorff dimensions of the spacetimes and their time slices, respectively. Remarkably, they, too, find that the (spectral) dimension $\mathcal{D}_\mathrm{s}$ of the spacetime reduces dynamically from $\mathcal{D}_\mathrm{s} \approx 4$ at large distances to $\mathcal{D}_\mathrm{s} \approx 2$ on small length scales.

While until recently it has been difficult to compare the continuum theory to the discrete causal triangulation approach, the new Monte Carlo results suggest that they might be closely related, possibly representing the same "universality class".

In our original argument [3] we determined the effective dimensionality of the fractal realized at sub-planckian distances (in the asymptotic scaling regime of the NGFP) from the anomalous dimension η_N at the NGFP. A priori this definition of an effective dimensionality is different from the one used in the Monte Carlo simulations. It is the main purpose of the present paper to apply the reasoning from [3, 5] to the definition of the effective dimension which was employed by Ambjørn, Jurkiewicz and Loll [36], namely the spectral dimension $\mathcal{D}_\mathrm{s}$.

We shall demonstrate that asymptotically safe QEG does indeed predict $\mathcal{D}_\mathrm{s} = 4$ at $\ell \gg \ell_\mathrm{Pl}$ and $\mathcal{D}_s = 2$ for $\ell \ll \ell_\mathrm{Pl}$. (The Planck length and mass are defined as $\ell_\mathrm{Pl} \equiv m_\mathrm{Pl}^{-1} \equiv G(k=0)^{1/2}$.)

As a preparation we review and extend the discussion of refs. [3, 5] in Section 2, and in Section 3 we compute the spectral dimension of the QEG spacetimes.

2. QEG spacetimes under the microscope

For simplicity we use the Einstein-Hilbert truncation to start with, and we consider spacetimes with classical dimensionality $d = 4$. The corresponding RG trajectories were completely classified and determined numerically in [4]. The physically relevant ones, for $k \to \infty$, all approach the NGFP at (g_*, λ_*) so that the dimensionful quantities run according to

$$G_k \approx \frac{g_*}{k^2}\,, \qquad \bar{\lambda}_k \approx \lambda_*\, k^2 \tag{2.1}$$

The behavior (2.1) is realized in the asymptotic scaling regime $k \gg m_\mathrm{Pl}$. Near $k = m_\mathrm{Pl}$ the trajectories cross over towards the gaussian fixed point at $g = \lambda = 0$, and then run towards negative, vanishing, and positive values of λ, respectively.

Since in this paper we are interested only in the limiting cases of very small and very large distances the following caricature of a RG trajectory will be sufficient. We assume that G_k and $\bar{\lambda}_k$ behave as in (2.1) for $k \gg m_\mathrm{Pl}$, and that they assume constant values for $k \ll m_\mathrm{Pl}$. The precise interpolation between the two regimes could be obtained numerically [4] but will not be needed here.

The argument of ref. [5] concerning the fractal nature of the QEG spacetimes was as follows. Within the Einstein-Hilbert truncation of theory space, the effective field equations (1.1) happen to coincide with the ordinary Einstein equation, but with G_k and $\bar{\lambda}_k$ replacing the classical constants. Without matter,

$$R_{\mu\nu}(\langle g\rangle_k) = \bar{\lambda}_k \, \langle g_{\mu\nu}\rangle_k \,. \tag{2.2}$$

Since in absence of dimensionful constants of integration $\bar{\lambda}_k$ is the only quantity in this equation which sets a scale, every solution to (2.2) has a typical radius of curvature $r_c(k) \propto 1/\sqrt{\bar{\lambda}_k}$. (For instance, the maximally symmetric S^4-solution has the radius $r_c = r = \sqrt{3/\bar{\lambda}_k}$.) If we want to explore the spacetime structure at a fixed length scale ℓ we should use the action $\Gamma_k[g_{\mu\nu}]$ at $k = 1/\ell$ because with this functional a tree level analysis is sufficient to describe the essential physics at this scale, including the relevant quantum effects. Hence, when we observe the spacetime with a microscope of resolution ℓ, we will see an average radius of curvature given by $r_c(\ell) \equiv r_c(k = 1/\ell)$. Once ℓ is smaller than the Planck length $\ell_{\rm Pl} \equiv m_{\rm Pl}^{-1}$ we are in the fixed point regime where $\bar{\lambda}_k \propto k^2$ so that $r_c(k) \propto 1/k$, or

$$r_c(\ell) \propto \ell \,. \tag{2.3}$$

Thus, when we look at the structure of spacetime with a microscope of resolution $\ell \ll \ell_{\rm Pl}$, the average radius of curvature which we measure is proportional to the resolution itself. If we want to probe finer details and decrease ℓ we automatically decrease r_c and hence *in*crease the average curvature. Spacetime seems to be more strongly curved at small distances than at larger ones. The scale-free relation (2.3) suggests that at distances below the Planck length the QEG spacetime is a special kind of fractal with a self-similar structure. It has no intrinsic scale because in the fractal regime, i.e. when the RG trajectory is still close to the NGFP, the parameters which usually set the scales of the gravitational interaction, G and $\bar{\lambda}$, are not yet "frozen out". This happens only later on, somewhere half way between the non-gaussian and the gaussian fixed point, at a scale of the order of $m_{\rm Pl}$.

Below this scale, G_k and $\bar{\lambda}_k$ stop running and, as a result, $r_c(k)$ becomes independent of k so that $r_c(\ell) = \text{const}$ for $\ell \gg \ell_{\rm Pl}$. In this regime $\langle g_{\mu\nu}\rangle_k$ is k-independent, indicating that the macroscopic spacetime is describable by a single smooth, classical riemannian manifold.

The above argument made essential use of the proportionality $\ell \propto 1/k$. In the fixed point regime it follows trivially from the fact that there exist no relevant dimensionful parameters so that $1/k$ is the only length scale one can form. The algorithm for the determination of $\ell(k)$ described in the Introduction yields the same answer.

It is easy to make the k-dependence of $\langle g_{\mu\nu}\rangle_k$ explicit. Picking an arbitrary reference scale k_0 we may rewrite (2.2) as $[\bar{\lambda}_{k_0}/\bar{\lambda}_k]\, R^\mu{}_\nu(\langle g\rangle_k) = \bar{\lambda}_{k_0}\, \delta^\mu_\nu$. Since $R^\mu{}_\nu(c\, g) = c^{-1}\, R^\mu{}_\nu(g)$ for any constant $c > 0$, this relation implies that the average metric scales as

$$\langle g_{\mu\nu}(x)\rangle_k = \frac{\bar{\lambda}_{k_0}}{\bar{\lambda}_k} \, \langle g_{\mu\nu}(x)\rangle_{k_0} \tag{2.4}$$

and its inverse according to

$$\langle g^{\mu\nu}(x)\rangle_k = \frac{\bar{\lambda}_k}{\bar{\lambda}_{k_0}} \langle g^{\mu\nu}(x)\rangle_{k_0} \,. \tag{2.5}$$

These relations are valid provided the family of solutions considered exists for all scales between k_0 and k, and $\bar{\lambda}_k$ has the same sign always.

As we discussed in ref. [3] the QEG spacetime has an effective dimensionality which is k-dependent and hence noninteger in general. Our discussion in [3] was based upon the anomalous dimension η_N of the operator $\int \sqrt{g}\, R$. It is defined as $\eta_N \equiv -k\,\partial_k \ln Z_{Nk}$ where $Z_{Nk} \propto 1/G_k$ is the wavefunction renormalization of the metric [1]. In a sense which we shall make more precise in a moment, the effective dimensionality of spacetime equals $4 + \eta_N$. The RG trajectories of the Einstein-Hilbert truncation (within its domain of validity) have $\eta_N \approx 0$ for $k \to 0$[1] and $\eta_N \approx -2$ for $k \to \infty$, the smooth change by two units occuring near $k \approx m_{\rm Pl}$. As a consequence, the effective dimensionality is 4 for $\ell \gg \ell_{\rm Pl}$ and 2 for $\ell \ll \ell_{\rm Pl}$.

In fact, the UV fixed point has an anomalous dimension $\eta \equiv \eta_N(g_*, \lambda_*) = -2$. We can use this information in order to determine the momentum dependence of the dressed graviton propagator for momenta $p^2 \gg m_{\rm Pl}^2$. Expanding the Γ_k of (1.2) about flat space and omitting the standard tensor structures we find the inverse propagator $\widetilde{\mathcal{G}}_k(p)^{-1} \propto Z_N(k)\, p^2$. The conventional dressed propagator $\widetilde{\mathcal{G}}(p)$ contained in $\Gamma \equiv \Gamma_{k=0}$ obtains from the *exact* $\widetilde{\mathcal{G}}_k$ in the limit $k \to 0$. For $p^2 > k^2 \gg m_{\rm Pl}^2$ the actual cutoff scale is the physical momentum p^2 itself[2] so that the k-evolution of $\widetilde{\mathcal{G}}_k(p)$ stops at the threshold $k = \sqrt{p^2}$. Therefore

$$\widetilde{\mathcal{G}}(p)^{-1} \propto\; Z_N\left(k = \sqrt{p^2}\right) p^2 \propto\; (p^2)^{1-\eta/2} \tag{2.6}$$

because $Z_N(k) \propto k^{-\eta}$ when $\eta \equiv -\partial_t \ln Z_N$ is (approximately) constant. In d dimensions, and for $\eta \neq 2-d$, the Fourier transform of $\widetilde{\mathcal{G}}(p) \propto 1/(p^2)^{1-\eta/2}$ yields the following propagator in position space:

$$\mathcal{G}(x;y) \propto\; \frac{1}{|x-y|^{d-2+\eta}} \,. \tag{2.7}$$

This form of the propagator is well known from the theory of critical phenomena, for instance. (In the latter case it applies to large distances.) Eq. (2.7) is not valid directly at the NGFP. For $d = 4$ and $\eta = -2$ the dressed propagator is $\widetilde{\mathcal{G}}(p) = 1/p^4$ which has the following representation in position space:

$$\mathcal{G}(x;y) = -\frac{1}{8\pi^2} \ln\left(\mu\,|x-y|\right) \,. \tag{2.8}$$

Here μ is an arbitrary constant with the dimension of a mass. Obviously (2.8) has the same form as a $1/p^2$-propagator in 2 dimensions.

[1] In the case of type IIIa trajectories [4, 31] the macroscopic k-value is still far above $k_{\rm term}$, i.e. in the "GR regime" described in [31].

[2] See ref. [29, section 1] for a detailed discussion of "decoupling" phenomena of this kind.

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Slightly away from the NGFP, before other physical scales intervene, the propagator is of the familiar type (2.7) which shows that the quantity η_N has the standard interpretation of an anomalous dimension in the sense that fluctuation effects modify the decay properties of $\mathcal{G}$ so as to correspond to a spacetime of effective dimensionality $4 + \eta_N$.

Thus the properties of the RG trajectories imply a remarkable dimensional reduction: spacetime, probed by a "graviton" with $p^2 \ll m_{\rm Pl}^2$ is 4-dimensional, but it appears to be 2-dimensional for a graviton with $p^2 \gg m_{\rm Pl}^2$ [3].

It is interesting to note that in d classical dimensions, where the macroscopic spacetime is d-dimensional, the anomalous dimension at the fixed point is $\eta = 2 - d$. Therefore, for any d, the dimensionality of the fractal as implied by η_N is $d + \eta = 2$ [3, 5].

3. The spectral dimension

In this section we determine the spectral dimension $\mathcal{D}_{\rm s}$ of the QEG spacetimes. This particular definition of a fractal dimension is borrowed from the theory of diffusion processes on fractals [37] and is easily adapted to the quantum gravity context [38, 36]. In particular it has been used in the Monte Carlo studies mentioned in the Introduction.

Let us study the diffusion of a scalar test particle on a d-dimensional classical Euclidean manifold with a fixed smooth metric $g_{\mu\nu}(x)$. The corresponding heat-kernel $K_g(x, x'; T)$ giving the probability for the particle to diffuse from x' to x during the fictitious diffusion time T satisfies the heat equation

$$\partial_T K_g(x, x'; T) = \Delta_g K_g(x, x'; T)\,, \tag{3.1}$$

where $\Delta_g \equiv D^2$ denotes the scalar laplacian: $\Delta_g \phi \equiv g^{-1/2}\,\partial_\mu(g^{1/2}\,g^{\mu\nu}\,\partial_\nu\phi)$. The heat-kernel is a matrix element of the operator $\exp(T\,\Delta_g)$. In the random walk picture its trace per unit volume,

$$P_g(T) \equiv V^{-1} \int d^d x\,\sqrt{g(x)}\,K_g(x, x; T) \;\;\equiv\;\; V^{-1}\,{\rm Tr}\,\exp(T\,\Delta_g)\,, \tag{3.2}$$

has the interpretation of an average return probability. (Here $V \equiv \int d^d x\,\sqrt{g}$ denotes the total volume.) It is well known that P_g possesses an asymptotic expansion (for $T \to 0$) of the form $P_g(T) = (4\pi T)^{-d/2} \sum_{n=0}^{\infty} A_n\,T^n$. For an infinite flat space, for instance, it reads $P_g(T) = (4\pi T)^{-d/2}$ for all T. Thus, knowing the function P_g, one can recover the dimensionality of the target manifold as the T-independent logarithmic derivative

$$d = -2\frac{d\ln P_g(T)}{d\ln T}\,. \tag{3.3}$$

This formula can also be used for curved spaces and spaces with finite volume V provided T is not taken too large [36].

In QEG where we functionally integrate over all metrics it is natural to replace $P_g(T)$ by its expectation value. Symbolically,

$$P(T) \equiv \langle P_\gamma(T)\rangle \equiv \int \mathcal{D}\gamma\,\mathcal{D}C\,\mathcal{D}\bar{C}\;P_\gamma(T)\;\,e^{-S_{\rm bare}[\gamma, C, \bar{C}]} \tag{3.4}$$

Here $\gamma_{\mu\nu}$ denotes the microscopic metric and S_{bare} is the bare action with the gauge fixing terms and the pieces containing the ghosts C and $\bar{C}$ included. Note that (3.4) does not contain any IR cutoff; it is the ordinary ($k = 0$) expectation value with all modes integrated out. In QEG the functional S_{bare} is given by the fixed point action. Given $P(T)$, the spectral dimension of the quantum spacetime is defined in analogy with (3.3):

$$\mathcal{D}_{\text{s}} = -2 \frac{d \ln P(T)}{d \ln T} . \tag{3.5}$$

Let us now evaluate the expectation value (3.4) using the average action method. The fictitious diffusion process takes place on a "manifold" which, at every fixed scale, is described by a smooth riemannian metric $\langle g_{\mu\nu} \rangle_k$. While the situation appears to be classical at fixed k, nonclassical features emerge in the regime with nontrivial RG running since there the metric depends on the scale at which the spacetime structure is probed.

The nonclassical features are encoded in the properties of the diffusion operator. Denoting the covariant laplacians corresponding to the metrics $\langle g_{\mu\nu} \rangle_k$ and $\langle g_{\mu\nu} \rangle_{k_0}$ by $\Delta(k)$ and $\Delta(k_0)$, respectively, eqs. (2.4) and (2.5) imply that they are related by

$$\Delta(k) = \frac{\bar{\lambda}_k}{\bar{\lambda}_{k_0}} \Delta(k_0) . \tag{3.6}$$

When $k, k_0 \gg m_{\text{Pl}}$ we have, for example,

$$\Delta(k) = \left(\frac{k}{k_0} \right)^2 \Delta(k_0) . \tag{3.7}$$

Recalling that the average action Γ_k defines an effective field theory at the scale k we have that $\langle \mathcal{O}(\gamma_{\mu\nu}) \rangle \approx \mathcal{O}(\langle g_{\mu\nu} \rangle_k)$ if the operator $\mathcal{O}$ involves typical covariant momenta of the order k and $\langle g_{\mu\nu} \rangle_k$ solves eq. (1.1). In the following we exploit this relationship for the RHS of the diffusion equation, $\mathcal{O} = \Delta_\gamma \, K_\gamma(x, x'; T)$. It is crucial here to correctly identify the relevant scale k.

If the diffusion process involves (approximately) only a small interval of scales near k over which $\bar{\lambda}_k$ does not change much the corresponding heat equation contains the $\Delta(k)$ for this specific, fixed value of k:

$$\partial_T K(x, x'; T) = \Delta(k) K(x, x'; T) . \tag{3.8}$$

Denoting the eigenvalues of $-\Delta(k_0)$ by $\mathcal{E}_n$ and the corresponding eigenfunctions by ϕ_n, this equation is solved by

$$K(x, x'; T) = \sum_n \phi_n(x) \, \phi_n(x') \, \exp\left(-F(k^2) \, \mathcal{E}_n \, T \right) . \tag{3.9}$$

Here we introduced the convenient notation $F(k^2) \equiv \bar{\lambda}_k / \bar{\lambda}_{k_0}$. Knowing this propagation kernel we can time-evolve any initial probability distribution $p(x; 0)$ according to $p(x; T) = \int d^4x' \sqrt{g_0(x')} \, K(x, x'; T) \, p(x'; 0)$ with g_0 the determinant of $\langle g_{\mu\nu} \rangle_{k_0}$. If the initial distribution has an eigenfunction expansion of the form $p(x; 0) = \sum_n C_n \, \phi_n(x)$ we obtain

$$p(x;T) = \sum_n C_n\, \phi_n(x)\, \exp\left(-F(k^2)\, \mathcal{E}_n\, T\right). \tag{3.10}$$

If the C_n's are significantly different from zero only for a single eigenvalue $\mathcal{E}_N$, we are dealing with a single-scale problem. In the usual spirit of effective field theories we would then identify $k^2 = \mathcal{E}_N$ as the relevant scale at which the running couplings are to be evaluated.

However, in general the C_n's are different from zero over a wide range of eigenvalues. In this case we face a multiscale problem where different modes ϕ_n probe the spacetime on different length scales.

If $\Delta(k_0)$ corresponds to flat space, say, the eigenfunctions $\phi_n \equiv \phi_p$ are plane waves with momentum p^μ, and they resolve structures on a length scale ℓ of order $1/|p|$. Hence, in terms of the eigenvalue $\mathcal{E}_n \equiv \mathcal{E}_p = p^2$ the resolution is $\ell \approx 1/\sqrt{\mathcal{E}_n}$. This suggests that when the manifold is probed by a mode with eigenvalue $\mathcal{E}_n$ it "sees" the metric $\langle g_{\mu\nu}\rangle_k$ for the scale $k = \sqrt{\mathcal{E}_n}$. Actually the identification $k = \sqrt{\mathcal{E}_n}$ is correct also for curved space since, in the construction of Γ_k, the parameter k is introduced precisely as a cutoff in the spectrum of the covariant laplacian.

Therefore we conclude that under the spectral sum of (3.10) we must use the scale $k^2 = \mathcal{E}_n$ which depends explicitly on the resolving power of the corresponding mode. Likewise, in eq. (3.9), $F(k^2)$ is to be interpreted as $F(\mathcal{E}_n)$. Thus we obtain the traced propagation kernel

$$\begin{aligned} P(T) &= V^{-1} \sum_n \exp\left[-F(\mathcal{E}_n)\, \mathcal{E}_n\, T\right] \\ &= V^{-1}\, \mathrm{Tr}\, \exp\left[F\Big(-\Delta(k_0)\Big)\, \Delta(k_0)\, T\right] \end{aligned} \tag{3.11}$$

It is convenient to choose k_0 as a macroscopic scale in a regime where there are no strong RG effects any more.

Furthermore, let us assume for a moment that at k_0 the cosmological constant is tiny, $\bar\lambda_{k_0} \approx 0$, so that $\langle g_{\mu\nu}\rangle_{k_0}$ is an approximately flat metric. In this case the trace in eq. (3.11) is easily evaluated in a plane wave basis:

$$P(T) = \int \frac{d^4p}{(2\pi)^4}\, \exp\left[-p^2\, F(p^2)\, T\right]. \tag{3.12}$$

The T-dependence of (3.12) determines the fractal dimensionality of spacetime via (3.5). In the limits $T \to \infty$ and $T \to 0$ where the random walks probe very large and small distances, respectively, we obtain the dimensionalities corresponding to the largest and smallest length scales possible. The limits $T \to \infty$ and $T \to 0$ of $P(T)$ are determined by the behavior of $F(p^2) \equiv \bar\lambda(k = \sqrt{p^2})/\bar\lambda_{k_0}$ for $p^2 \to 0$ and $p^2 \to \infty$, respectively.

For a RG trajectory where the renormalization effects stop below some threshold we have $F(p^2 \to 0) = 1$. In this case (3.12) yields $P(T) \propto 1/T^2$, and we conclude that the macroscopic spectral dimension is $\mathcal{D}_\mathrm{s} = 4$.

In the fixed point regime we have $\bar\lambda_k \propto k^2$, and therefore $F(p^2) \propto p^2$. As a result, the exponent in (3.12) is proportional to p^4 now. This implies the $T \to 0-$behavior $P(T) \propto 1/T$. It corresponds to the spectral dimension $\mathcal{D}_\mathrm{s} = 2$.

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This result holds for all RG trajectories since only the fixed point properties were used. In particular it is independent of $\bar{\lambda}_{k_0}$ on macroscopic scales. In fact, the above assumption that $\langle g_{\mu\nu}\rangle_{k_0}$ is flat was not necessary for obtaining $\mathcal{D}_{\rm s} = 2$. This follows from the fact that even for a curved metric the spectral sum (3.11) can be represented by an Euler-Mac Laurin series which always implies (3.12) as the leading term for $T \to 0$.

Thus we may conclude that on very small and very large length scales the spectral dimensions of the QEG spacetimes are

$$\begin{aligned} \mathcal{D}_{\rm s}(T \to \infty) &= 4 \\ \mathcal{D}_{\rm s}(T \to 0) &= 2\,. \end{aligned} \tag{3.13}$$

The dimensionality of the fractal realized at sub-planckian distances is found to be 2 again. It is by no means trivial that $\mathcal{D}_{\rm s}$ coincides with the value of $4+\eta$. While the replacement of the classical $p^2\,F(p^2) = p^2$ by $p^2\,F(p^2) \propto p^4$ is reminiscent of the graviton propagator argument of Section 2, it is important to emphasize that the value of $4+\eta$ is entirely determined by the running of G_k, while the spectral dimension was derived from the k-dependence of the cosmological constant.

In fact, it is remarkable that the equality of $4+\eta$ and $\mathcal{D}_{\rm s}$ is a special feature of 4 classical dimensions. Generalizing for d classical dimensions, the fixed point running of Newton's constant becomes $G_k \propto k^{2-d}$ with a dimension-dependent exponent, while $\bar{\lambda}_k \propto k^2$ continues to have a quadratic k-dependence. As a result, the $\widetilde{\mathcal{G}}(k)$ of eq. (2.6) is proportional to $1/p^d$ in general so that, for any d, the 2-dimensional looking graviton propagator (2.8) is obtained. (This is equivalent to saying that $\eta = 2-d$, or $d+\eta = 2$, for arbitrary d.)

On the other hand, the impact of the RG effects on the diffusion process is to replace the operator Δ by Δ^2, for any d, since the cosmological constant always runs quadratically. Hence, in the fixed point regime, eq. (3.12) becomes

$$P(T) \propto \int d^dp\,\exp\left[-p^4\,T\right] \propto T^{-\frac{d}{4}}\,. \tag{3.14}$$

This T-dependence implies the spectral dimension

$$\mathcal{D}_{\rm s}(d) = \frac{d}{2}\,. \tag{3.15}$$

This value coincides with $d+\eta$ if, and only if, $d = 4$. It is an intriguing speculation that this could have something to do with the observed macroscopic dimensionality of spacetime.

Up to this point, to be as concrete as possible, we formulated our argument within the Einstein-Hilbert truncation. To complete the discussion we show that the exact (untruncated) theory if it has a NGFP implies the dynamical dimensional reduction from 4 to 2 dimensions (in $d = 4$) in exactly the same way as the truncated one.

The complete effective average action has the structure $\Gamma_k[g_{\mu\nu}] = \sum_n \bar{\rm g}_n(k)\,I_n[g_{\mu\nu}]$ with infinitely many running couplings $\bar{\rm g}_n(k)$ and diffeomorphism invariant functionals I_n. If $\bar{\rm g}_n(k)$ has the canonical dimension d_n the corresponding dimensionless couplings are

$\mathrm{g}_n(k) \equiv k^{-d_n}\, \bar{\mathrm{g}}_n(k)$ and we have

$$\Gamma_k[g_{\mu\nu}] = \sum_n \mathrm{g}_n(k)\, k^{d_n}\, I_n[g_{\mu\nu}] \;=\; \sum_n \mathrm{g}_n(k)\, I_n[k^2\, g_{\mu\nu}]\,. \tag{3.16}$$

In the second equality we used that $I_n[c^2\, g_{\mu\nu}] = c^{d_n}\, I_n[g_{\mu\nu}]$ for any $c > 0$ since I_n has dimension $-d_n$.

If the theory is asymptotically safe at the exact level, all $\mathrm{g}_n(k)$ approach constant values g_{n*} for $k \to \infty$:

$$\Gamma_{k\to\infty}[g_{\mu\nu}] = \sum_n \mathrm{g}_{n*}\, I_n[k^2\, g_{\mu\nu}] \tag{3.17}$$

Obviously this functional depends on k^2 and $g_{\mu\nu}$ only via the combination $k^2\, g_{\mu\nu}$. Therefore the solutions of the corresponding field equation, $\langle g_{\mu\nu}\rangle_k$, scale proportional to k^{-2}. Hence $\Delta(k) \propto k^2$ in the fixed point regime, and this is exactly the scaling behavior (3.7) our above derivation of $\mathcal{D}_\mathrm{s}(T \to 0) = 2$ was based upon.

This completes the demonstration that if a NGFP does exist in the full theory, its exact spacetimes are fractals with $\mathcal{D}_\mathrm{s} = 2$ on sub-planckian distances.

At this point it is tempting to compare the result (3.13) to the spectral dimensions of the spacetime which were recently obtained by Monte Carlo simulations of the causal dynamical triangulation model [36]:

$$\begin{aligned} \mathcal{D}_\mathrm{s}(T \to \infty) &= 4.02 \pm 0.1 \\ \mathcal{D}_\mathrm{s}(T \to 0) &= 1.80 \pm 0.25\,. \end{aligned} \tag{3.18}$$

These figures, too, suggest that the long-distance and short-distance spectral dimension should be 4 and 2, respectively.

The dimensional reduction from 4 to 2 dimensions is a highly nontrivial dynamical phenomenon which seems to occur in both QEG and the discrete triangulation model. We find it quite remarkable that the discrete and the continuum approach lead to essentially identical conclusions in this respect. We consider this agreement a first hint indicating that (at least in 4 dimensions) the discrete model and QEG in the average action formulation might describe the same physics. But clearly much more work is needed in order to understand how the two approaches are related precisely.

4. Summary

The general picture of the spacetime structure in QEG which has emerged so far is as follows. At sub-planckian distances spacetime is a fractal of dimensionality $\mathcal{D}_\mathrm{s} = 4+\eta = 2$. It can be thought of as a self-similar hierarchy of superimposed riemannian manifolds of any curvature. As one considers larger length scales where the RG running of the gravitational parameters comes to a halt, the "ripples" in the spacetime gradually disappear and the structure of a classical 4-dimensional manifold is recovered.

Within the Einstein-Hilbert approximation of QEG there are two natural ways of defining an effective dimensionality of the fractal spacetime. We can either define it as $4+\eta_N$ as derived from the running of Newton's constant, or we use the spectral dimension implied by the k-dependence of the cosmological constant. We have seen that both definitions lead to identical results on very large and very small distances. We also showed that the microscopic dimensionality $\mathcal{D}_{\mathrm{s}} = 2$ is a rather direct and exact consequence of asymptotic safety which does not rely on any approximation or truncation. It is therefore not unlikely that the mechanism of a dynamical dimensional reduction from 4 to 2 dimensions which occurs in QEG is the same phenomenon as the dimensional reduction observed in the Monte Carlo studies of causal dynamical triangulations.

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